# TEACHER'S RESOURCE BOOK

# Number Facts 1

AND

# Number Facts 2

## A New Approach to Tables

The Educational Company of Ireland

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# 1. Introduction

***Number Facts*** is a series of activity books designed to foster fluency in number facts (or 'tables') for primary school children from First Class. The series features an innovative approach to the acquisition of basic number facts, and teaches children to *understand*, not just *do*, maths.

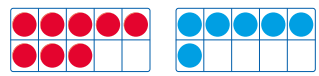
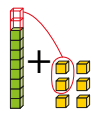
Traditionally, learning tables has been by rote, but current research suggests that this is ineffective for the majority of children. In contrast, ***Number Facts*** teaches children to visualise numbers and to use images and thinking strategies *to use what they know to solve what they do not know.*

In contrast to the more traditional drill-and-practice workbooks, which just test whether the answer is known, ***Number Facts*** teaches children to visualise numbers pictorially and to use these images and **thinking strategies** to become more adept at manipulating numbers.

A **thinking strategy** is a way to think about a process to arrive efficiently at an answer (see Thinking Strategies Overview, p. 11) . For example, if asked to add 9 to a number, one could think to go on 10 and then to go back 1.

The specific focus of ***Number Facts*** will be to develop children's **thinking strategies** and apply these to the basic number facts in such a way as to promote the child's ability to visualise and recall these facts, thereby achieving fluency.

To achieve fluency, children must be accurate, efficient and flexible:

- **Accurate:** children must arrive at the correct answer, e.g. 8 + 6 = 14.
- **Efficient:** children must work efficiently. A child who produces an answer of 14 in response to the question 8 + 6 by 'counting on' (mentally counting in jumps of one, or using counters, fingers, etc.) may be accurate but is not efficient.
- **Flexible:** children must be able to visualise and mentally manipulate numbers in order to see how they might be broken down and recombined to get an accurate and efficient answer, e.g.
  - 8 + 6 = (5 + 3) + (5 + 1) = 10 + 4 (make a ten) = 14 
  - 8 + 6 = 10 + 4 (compensation / make a ten) = 14 
  - 8 + 6 = 7 + 7 (move one to make doubles) = 14 

Flexibility is the key to fluency. A child who is flexible with number facts is one with a well-developed number sense who can see the connections both between and within numbers, i.e. they can partition and/or combine numbers into more compatible (friendly) amounts and can apply their strategies to numbers beyond those they have dealt with. Therefore, while learning tables was traditionally emphasised for numbers up to 10 + 10, the thinking strategies approach used in this series of books enables children to apply these mental computation skills to numbers beyond the traditional ceiling of 10 + 10, e.g. 23 + 9, 46 + 9. The **Challenge** sections typically include this type of calculation, in order to provide the children with opportunities to apply thinking strategies to quantities outside the usual range (for more on the Challenge sections, see p. 9).

**Digital Resources**
Visit **www.edcolearning.ie** to access:
- a comprehensive list of weblinks for this book
- additional long-term plans.

# 2. The *Number Facts* Approach

## Concrete–pictorial–abstract approach

The ***Number Facts*** series is based on a concrete–pictorial–abstract (CPA) approach (for more information on this approach, please see the Weblinks document at www.edcolearning.ie), and the activities in each unit should progress from the concrete to the pictorial to the abstract.

### Concrete activities

Concrete activities are those in which the children explore, pick, move and place real objects (such as ten frames and counters, number racks or rekenreks, base ten blocks, dominoes, etc.) While the practicalities of working with real objects will likely mean that these activities will take time to complete, these activities are very important at the introductory stage when the foundations are laid for the children to fully understand the abstract concepts.

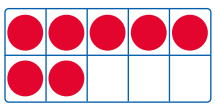
- These exploratory activities will be most beneficial if completed in class a week or so prior to the children starting the unit in their ***Number Facts*** book. In this way, the activities in the book serve as consolidation.
- These activities can also be very effective when used as the basis for an instructional station as part of team teaching.
- Since the majority of the concrete activities require the children to use ten frames  , it is recommended that each child has their own ten frame and counters (see Photocopiable 1, Ten Frames and Counters, p. 49). Alternatively, those who use *Operation Maths* could use the twenty frame provided with that series.
- The concrete activities in each unit can be completed in class or for homework if the book is intended primarily for home use.

### Pictorial activities

- Pictorial activities may comprise visual representations of concrete materials such as base ten blocks, or ten frames and counters; or something different such as dot grids, rainbow bonds or branching bonds. They act as a vital intermediary step in the progression from the concrete to the abstract. ***Number Facts 1*** and ***Number Facts 2*** primarily include the following pictorial activities:

#### *Ten frames*

Ten frames provide a pictorial representation of the concrete resource with which the children will likely be most familiar. Photocopiable 2, Ten-Frame Flash Cards (Sets 1 and 2, pp. 50–51) can also be used to help reinforce the visual structure of each number.

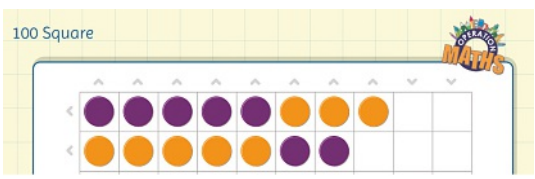


#### *Dot grids*

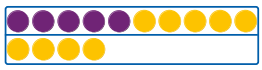
- Not to be confused with the ten frame, the dot grids are a pictorial representation of the beads on a rekenrek or number rack, where each row has 10 beads, with contrasting colours for each group of five beads.

8 + 7 = 15

- In a similar way to the ten frame, this structure reinforces the benchmark numbers of 5 and 10, thus 8 + 7 can be seen as (5 + 3) + (5 + 2) = 10 + 5. In a class where there is access to a rekenrek, it would be beneficial to use this during the initial concrete activities. Interactive digital versions can also be used (see the Weblinks document at www.edcolearning.ie). Alternatively, for those who use *Operation Maths,* the 100 Square eManipulative can be used to show the dot grids used.



- **Note:** in ***Number Facts***, when the dot grids represent two separate addends for addition number sentences, the dots are divided by a horizontal line. When representing a minuend (the starting quantity from which another amount is to be subtracted) greater than 10 in a subtraction number sentence, there is no horizontal dividing line.



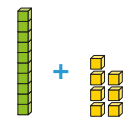
10 + 4 = 14

14 – 5 = 9

- Photocopiable 3, Dot Grids (p. 52), can be used when exploring how to combine or partition quantities using this model. Photocopiable 4, Dot-Grid Flash Cards (p. 53), can also be used to help reinforce the visual structure of each number.

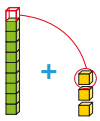
## Base ten blocks

- Also known as Dienes blocks, these are used to represent the addition and subtraction of 10, 9, 8 and 7. In the case of addition and subtraction of 10, these blocks emphasise that:
  10 + 7 = 1T + 7U = seventeen (the suffix '–teen' means 'and ten').



10 + 7 = 17

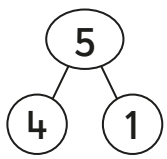
- For the addition of 9, 8 and 7, the adapted images emphasise how it is more efficient to think of 'making a ten' by taking the required number of units from one addend to combine with the other addend to make 10, which is a friendly/compatible number. This is the strategy of **compensation**, which will be developed and applied to bigger numbers in ***Number Facts 3*** and ***Number Facts 4***. Compensation as a strategy is also explored in *Operation Maths*. For more information on compensation, please see the Weblinks document at www.edcolearning.ie.



9 + 3 = 10 + 2 = 12

## Branching number bonds

Branching number bonds illustrate operations *without using the operational symbols* that can be confusing to children. The branching number bonds reinforce the idea that the top number (or the whole) can be broken down into different amounts (or parts), and, conversely, that the different amounts (or parts) can combine to make the top number (or whole). In this way, they can represent both addition and subtraction, and, as such, they are also an example of a part–whole model (see 'Bar models', below). For extra practice with branching number bonds, Photocopiable 5, Number Bonds (p. 54) can be used.

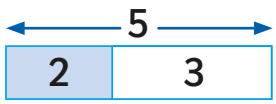



4 + 1 = 5
1 + 4 = 5
5 – 4 = 1
5 – 1 = 4

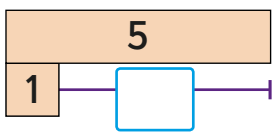
## Bar models

There are two types of bar model used in ***Number Facts 1*** and ***Number Facts 2***.

- **Part–whole bar model**: this model comprises a bar that represents the whole and which shows its constituent parts (see also 'Branching number bonds', above).

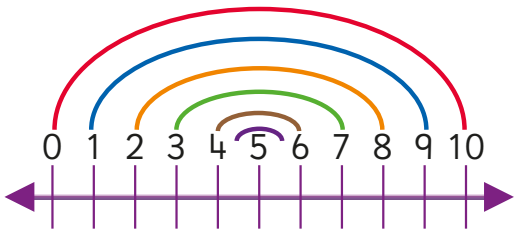



- **Comparison bar model** (***Number Facts 2*** only): this model is used to compare two amounts. In ***Number Facts 2*** it is used to elicit the difference between two amounts.




## Rainbow bonds

Rainbow bonds are based on a number line. The children join the numbers that combine to make a target number, e.g. 10, with the resultant shape being similar to a rainbow. This structure reinforces the fact that if 5 + 5 = 10, then the next closest numbers (one more or one less) must also equal the target number, i.e. 4 + 6 = 10.



***Number Facts 1*** and ***Number Facts 2*** also use other visuals for variety such as number lines (to demonstrate counting on or counting back, e.g. +/– 1, +/– 2, and to find difference), dominoes, and dice. Children should be encouraged to also use the 100 Square on the inside back cover of their books to discover patterns.

## Abstract activities

Abstract activities are those that use only digits and symbols or words, for example, a number sentence (6 + 4 = 10) or a word problem. While word problems are not included in this series, the children should be encouraged to apply the thinking strategies they learn in ***Number Facts*** when they encounter word problems in other situations. Early finishers and more able children could be encouraged to compose word problems to match a selection of number sentences they have solved.

### *Frame*

A frame ☐ is used to represent the unknown value in number sentences. Generally, the frame is located at the end or in the middle of the number sentences, although it is sometimes used in the initial position. Solving a number sentence with the frame in the initial position can be more difficult, so this is often reserved for Challenge sections or to add an extra dimension to a set of familiar facts.

### *Operations symbols (= / + / –) and language*

In ***Number Facts 1*** the equals sign is typically towards the end of the number sentence. In ***Number Facts 2*** it is also regularly located towards the beginning of the number sentence, for example 6 = 4 + ☐. This is to encourage children to become familiar with this positioning and to reinforce the idea that both sides of the equals sign should be the same (balanced).

Symbols like + and – can, in themselves, be very abstract. This is why the initial part of every unit uses words instead of symbols, both to promote language development alongside the development of thinking strategies and to reduce the potential for misunderstanding. Furthermore, as subtraction is a new operation in First Class, the symbol for subtraction (–) is not introduced until Unit 6 in ***Number Facts 1*** in order to allow for the children to have encountered it in their core maths programme.

| Operations Language | | |
|---|---|---|
| | ***Number Facts 1*** | ***Number Facts 2*** |
| **Addition** | and, add, plus | add, plus, sum of |
| **Subtraction** | take away, subtract | subtract, minus, difference between |

## Flexible approach

Above all, encourage flexibility. Prompt the children to explore and discuss alternative ways to solve the questions.

## Commutative property

While children should understand the commutative property of addition (e.g. that 8 + 6 = 6 + 8), in ***Number Facts*** they are also encouraged to think of different facts in a certain order.

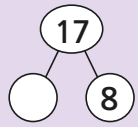
- For example, when adding 1 and 2, these facts are better presented as n + 1 or n + 2, as opposed to 1 + n or 2 + n, since the thinking strategy is to **count on**. In ***Number Facts 1*** and ***Number Facts 2*** the children will regularly be prompted to 'Count on from the biggest number.'
- However, when adding 10 it is more logical to think of 10 + n (when n is 0 to 9) as opposed to n + 10, since the former approach aligns the numbers with their place value positions.

## Inverse

In ***Number Facts 1*** and ***Number Facts 2*** addition and subtraction facts are generally taught in different units (with the exception of the units on Doubles and Near Doubles, as well as on In-between Doubles in ***Number Facts 2***). However, it is very important that the children are enabled to understand how these operations are the inverse of each other.

- Pictorial activities, such as branching number bonds and bar models, reinforce the inverse property, as they can be recorded using addition and subtraction sentences. If these activities are accompanied by a 'Think' box, this is usually emphasised in the text.
- The addition units cover subtraction as complementary addition, e.g. 3 + ☐ = 9.
- The branching number bonds and accompanying number sentences in the Revision sections are specifically included to reinforce the concept of addition and subtraction as inverse operations.

**Think:**
17 take away what number equals 8?



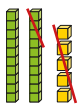

Or 8 plus what number equals 17?

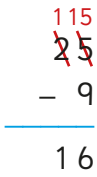
## Flexibility

When subtracting, the usual approach is to subtract from the units and then regroup if necessary.

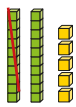
While that would be the correct procedure for written calculations (i.e. the 'column method'), for mental calculations it can be more efficient to think of taking from the tens.

**Written approach**






**Mental approach**



25 subtract 10 is 15,
so 25 subtract 9 is 16 (15 + 1).

However, if the above calculation was 25 subtract 1 (or 2, 3, 4 or 5), it would be more efficient to think of subtracting from the units.

The units on subtraction typically require the children to draw a strike through the artwork to show the subtrahend (the amount being subtracted). The children should be allowed to do this in any way they choose; i.e. they do not always have to follow the example that may have been given. In the Challenge sections of ***Number Facts 2***, examples have specifically not been given so as not to influence the approach.

The units on addition typically require the children to draw more counters, dots, cubes, etc. Again, the children should be allowed to draw them as they wish; this in turn might generate a discussion on the efficiency of one approach over another.

## Number Talks

***Number Facts*** is ideal for use with any class that is also participating in Number Talks. Number Talks are short, daily exercises aimed at building number sense. Children who have strong number sense can usually be flexible enough to solve calculations using a variety of thinking strategies. ***Number Facts*** greatly enhances a child's ability to verbalise and explain their strategy, even when applied to calculations outside 10 + 10. For more information on Number Talks please see the the Weblinks document at www.edcolearning.ie.

In short, the most efficient approach will always depend on the numbers involved, therefore flexibility is key.

# 3. Structure of the Books

The ***Number Facts*** series comprises weekly units that include activities for Monday to Thursday. These activities may be completed during maths class or assigned as homework activities. The sequence of the weekly units in each ***Number Facts*** book has been planned with due consideration to the following factors:

- There is a logical progression to the weekly units, so the series can be used in conjunction with any core maths programme.
- The series also serves as a companion to the *Operation Maths* programme, in that the units align (where suitable) to a relevant chapter or topic in *Operation Maths*. (Long-term plans that include both *Operation Maths* and *Number Facts* are available to download from www.edcolearning.ie.)
- The units are grouped together in logical blocks that align with the breaks that occur during the school year.
- For multi-class situations, the units in ***Number Facts 1*** typically align with the identical units in ***Number Facts 2*** in order to make it easier for the teacher to cover the same thinking strategy with both groups at the same time (see Long-term Plans, p. 12).

## Features

### Thinking strategies

Each unit has a specific underlying **thinking strategy** (see Thinking Strategies Overview, p. 11). The specific strategy is provided in the footer of the first page in each unit:

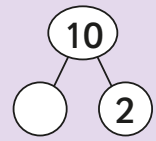
**Thinking Strategy** The various ways to partition 5 (benchmark number).

It is recommended that the thinking strategy is emphasised as often as possible during the unit. However, it is also important to encourage the children to consider or suggest if a different thinking strategy could also have been used, as this promotes flexible thinking.

### Think boxes

Think boxes are provided (usually on the second page of each unit) that serve as prompts in order to reinforce the relevant thinking strategies. The Think boxes may include a visual reminder (e.g. a branching number bond or part–whole bar model); a closed question (e.g. 6 take away what number equals 4?) or an open question (e.g. Think: which 'Take Away 4' facts are easiest? Why?). The open questions can be used to generate discussion and reflection among the class.

**Think:**
10 take away what number equals 2?




Or 2 plus what number equals 10?

### Challenge sections

Most days include a challenge section for the more able children. These sections typically include calculations that provide the children with opportunities to apply the relevant thinking strategy to quantities outside the usual range. It is recommended that the children are allowed to present the approaches they took to solve the challenges. While, as adults, we might have our own ideas of the 'right' way to solve these, the children should be encouraged to explore their own methods and to explain these to the class. (See also Number Talks, p. 8.)

**3 Challenge**

(a) 10 + 15 = ☐
(b) 10 + 13 = ☐
(c) 10 + 18 = ☐
(d) 10 + 16 = ☐

## Family card games

A selection of card games is provided for parents and children to play at home in order to reinforce the number facts taught in each unit. The suggested card game for each unit is included in the footer of the second page in each unit, as a 'Dear Family' note:

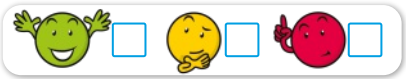
> **Dear Family** Play 'Make 5' (see page 70).

The games are explained on page 70 of the ***Number Facts*** books. Card games specifically were included, as a pack of cards is the most likely resource that a family would have at home. However, if these games are being played at school, they could be easily adapted for use with dice, e.g. roll the dice and double the number for **Doubles Snap**; roll the dice and add 10 for **Add Snap with 10**, etc.

## Revision and Assessment

### *Self-assessment*

The children can assess their own learning at the end of every week by ticking a box to represent their experience of that unit:



### *Revision*

A Revision section is included at the back of each ***Number Facts*** book. There are five Revision sections, which usually comprise ten number branching bonds. The children must complete each bond and then write four number sentences to match each bond. This reinforces the fact that four different number sentences can be written for each set of related numbers. The page footers indicate when children are ready to complete the relevant Revision section:

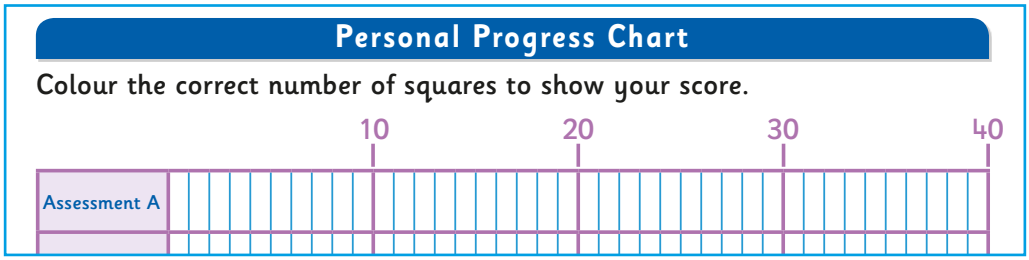
> You are now ready to do Revision A (page 58).

### *Assessment*

An Assessment section is included at the back of each ***Number Facts*** book. There are seven Assessments, which comprise four sets of ten questions (number sentences). There is a note included after each Revision section that directs the children to the relevant Assessment:

> You are now ready to do Assessment A (page 66).

The children can keep track of their assessment scores in their **Personal Progress Chart** on page 69 of each book.




**Note:** the Revision and Assessment activities do not have to be administered specifically at the time instructed in the footers (as outlined above); they can be administered at any stage after the relevant content has been covered. Assessments F and G are general assessments for completion at any point after the book content has been completed.

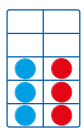
## Early finishers

If this book is intended for school rather than home use, it can be useful to have ideas ready for the early finishers:

- These children could be asked to write number sentences into their copies to match the pictorial elements included in that unit, such as branching bonds, rainbow bonds, etc.
- They could write word problems to match a selection of number sentences from the unit.
- They could play the suggested family card game in class.

# 4. Thinking Strategies Overview

A thinking strategy is a way to think about a process to arrive efficiently at an answer. The thinking strategies that children will learn in these books include:

| | |
|---|---|
| **(a) Number bonds of 5*, 10 and 20****<br>Through concrete and pictorial experiences, children are enabled to identify and remember the different pairs of numbers that combine to make up the numbers 5, 10 and 20. E.g. the number bonds of 10 include: 0 + 10; 1 + 9; 2 + 8, etc. | **(b) Adding and subtracting 1, 2 and 3**<br>Most young children can mentally count on and back one, two or three numbers, but counting on or back more than this can be difficult and inefficient. |
| **(c) Adding and subtracting 10**<br>Children are enabled to understand that the numbers 11 to 19 are made up of a ten and some units, e.g.<br>17 = 1 ten + 7 units = 10 + 7 = 17. | **(d) Adding and subtracting 9**<br>If 10 + 7 = 17, then 9 + 7 = 16 (one less). Similarly, if 17 – 10 = 7, then 17 – 9 = 8 (one more). Also, children can use compensation to make a ten, e.g. 9 + 7 = 10 + 6 = 16. |
| **(e) Adding and subtracting 8 and 7**<br>Similar to adding and subtracting 9, the children can compensate to make a ten, e.g.<br>8 + 7 → 10 + 5 (*move 2*) = 15.<br>7 + 6 → 10 + 3 (*move 3*) = 13. | **(f) Adding and subtracting 5**<br>The extensive use of a ten frame  enables children to visualise these facts. E.g. the number 9 displayed on a ten frame comprises 5 on the top and 4 on the bottom.  Therefore, 9 = 5 + 4. |
| **(g) Adding and subtracting 6 and 4**<br>If 5 + 4 = 9, then 6 + 4 = 10 (one more) and 4 + 4 = 8 (one less). | **(h) Adding and subtracting with zero**<br>When you add zero to or subtract zero from a number, the number does not change.<br>4 + 0 = **4**; 4 – 0 = **4** |
| **(i) Doubles and halves**<br>  <br>● Through concrete and pictorial experiences, children are enabled to identify doubles from their shapes. E.g. 3 + 3 = 6.<br>● Halves are the inverse of doubles, therefore if 6 + 6 = 12, then half of 12 = 6. | **(j) In-between doubles****<br>When shown using concrete materials, 6 + 8 can be changed into 7 + 7 by balancing out the counters. These can also be referred to as two-apart facts, as it can be done with all facts that differ by two, e.g. 5 + 7, 9 + 7, etc. |
| **(k) Near doubles**<br>If 6 + 6 = 12, then 6 + 7 = 13 (one more). Similarly, if 9 + 9 = 18, then 9 + 8 = 17 (one less). | **(l) Groups of ten**** The word names for the multiples of ten tell the number of groups of ten, e.g. sixty = six tens = 60. |

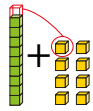
It is very important that children realise that there can often be many different thinking strategies for the same fact. For example, consider 9 + 8:

**Adding 10:** 10 + 8 = 18, so 9 + 8 = 17 (one less).

**Make a ten:** 9 + 8 is the same as 10 + 7 = 17.

**Doubles / Near doubles:** 9 + 9 =18, so 9 + 8 = 17.

**Doubles / Near doubles:** 8 + 8 = 16, so 9 + 8 = 17.



* For First Class only.
** For Second Class only.

# 5. Long-term Plans

| | *Number Facts 1* | *Number Facts 2* |
|---|---|---|
| **September** | 1. Number bonds of 5<br>2. Number bonds of 10<br>3. Doubles to 10<br>4. Add 1 | 1. Number bonds of 10<br>2. Add 1<br>3. Add 2<br>4. Subtract 1 |
| **October** | 5. Add 2<br>6. Take away 1<br>7. Take away 2 | 5. Subtract 2<br>6. Add 10<br>7. Subtract 10 |
| | **Revision A**<br>**Assessment A (Units 1–7)*** | |
| **November** | 8. Add 10<br>9. Take away 10<br>10. Zero<br>11. Doubles to 20 | 8. Zero<br>9. Doubles<br>10. Add 5<br>11. Subtract 5 |
| **December** | 12. Add 5<br>13. Take away 5 | 12. Add 9<br>13. Subtract 9 |
| | **Revision B**<br>**Assessment B (Units 8–13)** | |
| **January** | 14. Near doubles<br>15. Add 6<br>16. Take away 6<br>17. Add 4 | 14. Near doubles<br>15. Add 6<br>16. Subtract 6<br>17. Add 4 |
| **February** | 18. Take away 4<br>19. Add 9<br>20. Take away 9 | 18. Subtract 4<br>19. Number bonds of 20<br>20. In-between doubles |
| | **Revision C**<br>**Assessment C (Units 14–20)** | |
| **March** | 21. Add 8<br>22. Take away 8<br>23. Add 7<br>24. Take away 7 | 21. Add 8<br>22. Subtract 8<br>23. Add 7<br>24. Subtract 7 |
| **April** | 25. Add 3<br>26. Take away 3 | 25. Add 3<br>26. Subtract 3 |
| **May** | **Revision D**<br>**Assessment D (Units 21–26)** | |
| | 27. Number bonds of 20<br>28. Halves | 27. Halves<br>28. Groups of 10 |
| | **Revision E**<br>**Assessment E (Units 27–28)** | |
| **June** | **Assessment F (All units)**<br>**Assessment G (All units)** | |

*It is anticipated that the relevant Revision and Assessment sections can both be completed in one week.