## TEACHER'S RESOURCE BOOK

# Number Facts 3 

## AND

## Number Facts 4

## A New Approach to Tables

The Educational Company of Ireland

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Design and layout: Design Image
Proofreader: Jane Rogers
Illustrations: Design Image; Adam Linley (p. 11); Brian Fitzgerald (p. 54).
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## 1. Introduction

Number Facts is a series of activity books designed to foster fluency in number facts (or 'tables') for primary school children from First Class. The series features an innovative approach to the acquisition of basic number facts, and teaches children to understand, not just do, maths. Traditionally, learning tables has been by rote, but current research suggests that this is ineffective for the majority of children. In contrast, Number Facts teaches children to visualise numbers and to use images and thinking strategies to use what they know to solve what they do not know.

In contrast to the more traditional drill-and-practice workbooks, which just test whether the answer is known, Number Facts teaches children to visualise numbers pictorially and to use these images and thinking strategies to become more adept at manipulating numbers. A thinking strategy is a way to think about a process to arrive efficiently at an answer (see Thinking Strategies Overview, p. 12). For example, if asked to multiply a number by 5 , one could think of 10 times the number and then halve that amount.

The specific focus of Number Facts is to develop children's thinking strategies and apply these to the basic number facts in such a way as to promote the child's ability to visualise and recall these facts, thereby achieving fluency. To achieve fluency, children must be accurate, efficient and flexible:

- Accurate: children must arrive at the correct answer, e.g. $6 \times 8=48$.
- Efficient: children must work efficiently. A child who produces an answer of 48 in response to the question $6 \times 8$ by counting in jumps of six or eight may be accurate but is not efficient.
- Flexible: children must be able to visualise and mentally manipulate numbers in order to see how they might be broken down and recombined to get an accurate and efficient answer, e.g.

- $6 \times 8=8 \times 6$ (commutative property) $=$ double, double, double $6,-\ldots-{ }^{-}$ i.e. double 6 is 12 , double 12 is 24 , and double 24 is 48 .

Think: double, double, double!

| $?$ |  |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 24 |  | 24 |  |  |  |  |
| 12 | 12 | 12 | 12 |  |  |  |
| 6 | 6 | 6 | 6 | 6 | 6 | 6 |

Flexibility is the key to fluency. A child who is flexible with number facts is one with a well-developed number sense who can see the connections both between and within numbers, i.e. they can partition and/ or combine numbers and groups of numbers into more compatible (friendly) amounts and can apply their strategies to numbers beyond those they have dealt with. Therefore, while learning tables was traditionally emphasised for numbers up to $10 \times 10$, the thinking strategies approach used in this series of books enables children to apply these mental computation skills to numbers beyond the traditional ceiling of $10 \times 10$, e.g. $5 \times 18$ and $9 \times 20$. The Challenge sections (see p. 9) typically include this type of calculation, in order to provide the children with opportunities to apply thinking strategies to quantities outside the usual range.

## Digital Resources

Visit www.edcolearning.ie to access:

- a comprehensive list of weblinks for this book
- additional long-term plans for the series.


## 2. The Number Facts Approach

## Concrete-pictorial-abstract approach

The Number Facts series is based on a concrete-pictorial-abstract (CPA) approach (for more information on this approach, please see the Weblinks document at www.edcolearning.ie), and the activities in each unit should progress from the concrete to the pictorial to the abstract.

## Concrete activities

Concrete activities are those in which the children manipulate real objects (such as ten frames and counters, coins, marbles, base ten blocks, place value discs, etc.) While the practicalities of working with real objects will likely mean that these activities will take time to complete, these activities are very important at the introductory stage when the foundations are laid for the children to fully understand the abstract concepts.

- These exploratory activities will be most beneficial if completed in class a week or so prior to the children starting the unit in their Number Facts book. In this way, the activities in the book serve as consolidation.
- These activities can also be very effective when used as the basis for an instructional station as part of team teaching.
- The Number Facts books regularly prompt the children to 'use objects'. These can be any objects that are readily accessible, e.g. crayons, markers, pencils or paperclips. In school, counters or cubes could also be used; and at home, Lego pieces or spoons could be used.
- The concrete activities in each unit can be completed in class or for homework if the book is intended primarily for home use.


## Pictorial activities

Pictorial activities may comprise visual representations of concrete materials such as coins, counters, marbles or base ten blocks; or something different such as dot grids, branching number bonds, number lines or bar models. They act as a vital intermediary step in the progression from the concrete to the abstract. Number Facts $\mathbf{3}$ and Number Facts 4 primarily include the following pictorial activities:

## Dot grids

Dot grids are a pictorial representation of the beads on a rekenrek or number rack, where each row has ten beads, with contrasting colours for each group of five beads.

In a similar way to the ten frame, this structure reinforces the benchmark numbers of 5 and 10 , thus $7+7$ can be seen as $(5+2)+(5+2)=10+4$. In a class where there is access to a rekenrek, it would be beneficial to use this during the initial concrete activities. Interactive digital versions can also be used (see the Weblinks document at www.edcolearning.ie). Alternatively, for those who use Operation Maths, the 100 Square eManipulative can be used to display the dot grids used.

Note: in Number Facts, when a dot grid represents two separate addends for addition number sentences, the dots are divided by a horizontal line. When representing a minuend (the starting quantity from which another amount is to be subtracted) greater than 10 in a subtraction number sentence, there is no horizontal dividing line.

$7+7=(5+2)+(5+2)$
$=10+4$
$=14$


## 100 Dots grid

The 100 Dots grid is a larger, ten-row version of the dot grids above. Again, this is a pictorial representation of the beads on a ten-row rekenrek or number rack, where each row has ten beads, and contrasting colours are used for each group of five beads.

This structure reinforces the benchmark numbers of 5, 10 and 100, thus building

$4 \times 8=32$ on the children's experiences of ten frames and dot grids in Number Facts 1 and

## Number Facts 2.

The children can use the 100 Dots grid on the inside back cover of their books to show arrangements. To do this, divide an A4 page in half vertically and slide the resulting strips in and out to model various arrangements (as shown above right). Photocopiable 5, 100 Dots Grids (p. 56) can also be used to record various arrays of dots.

Again, in a class where there is access to a rekenrek it would be beneficial to use this during the initial concrete activities. Interactive digital versions can also be used (see the Weblinks document at www.edcolearning.ie), and those who use the Operation Maths series can model the questions in the Number Facts units using the 100 Square e Manipulative.


## Base ten blocks

Also known as Dienes blocks, base ten blocks are used to represent addition and subtraction. For addition using 9 and 8 , the adapted images emphasise how it is more efficient to think of 'making a ten' by taking the required number of units from one addend to combine with the other addend to make 10 , which is a friendly/compatible number. This is the strategy of compensation, which was


$$
9+3=10+2=12
$$ introduced initially in Number Facts 1 and Number Facts 2.

Compensation as a strategy is also developed further in Operation Maths. For more information on compensation, please see the Weblinks document at www.edcolearning.ie.

Photocopiable 1, Base Ten Blocks (p. 52) can be used to create representations of base ten blocks so that more children have access to them for modelling.

## Place value discs (10) (100)

An alternative to the base ten blocks, these can be used to represent amounts of units, tens, hundreds, and thousands. Photocopiable 2, Place Value Discs (p. 53) can be used to create similar discs.

## Branching number bonds

Branching number bonds illustrate operations without using the operational symbols that can be confusing to children.

- Addition and subtraction: these branching number bonds reinforce the idea that the top number (or the whole) can be broken down into different amounts (or parts), and, conversely, that the different amounts (or parts) can combine to make the top number (or whole). In this way, they can represent both addition and subtraction (e.g. $4+1=5 ; 5-4=1$ ). They can also be used to partition products and dividends into simpler, more compatible amounts. As such, they are also an example of a part-whole model (see 'Bar models', over).
- Multiplication and division: the same branching structure can also be used to show the factors that combine to create a product and can serve to illustrate the inverse relationship between multiplication and division (e.g. $8 \times 3=24$; $24 \div 8=3$ ).
- Compensation bonds: these bonds are a development of the usual branching bonds and are used to show how a quantity can be taken off one addend and combined with the other to make more friendly/compatible numbers and therefore be easier to calculate mentally. For extra practice with branching number bonds, Photocopiable 4, Number Bonds (p. 55) can be used.


## Bar models

There are two types of bar model used in Number Facts 3 and Number Facts 4.

- Part-whole bar model: this model comprises a bar that represents the whole and which shows its constituent parts (see also 'Branching number bonds', above). When representing multiplication and division, each part (or fraction of the overall bar) is equal in size.



## Think:

what number equals
4 groups of 5?


- Comparison bar model (Number Facts $\mathbf{3}$ only): this model is used to compare two amounts. In Number Facts $\mathbf{3}$ it is used to elicit the difference between two amounts.


Number Facts 3 and Number Facts 4 also use other visuals for variety such as number lines, 100 Square extracts, ten frames, place value grids, money and marbles.

## Abstract activities

Abstract activities are those that use only digits and symbols or words, for example, a number sentence ( $6 \times 4=$ $\square$ ) or a word problem. While word problems are not included in this series, the children should be encouraged to apply the thinking strategies they learn in Number Facts when they encounter word problems in other situations.

## Frame

A frame $\square$ is used to represent the unknown value in number sentences. The frame is not always located at the end of a number sentence, as would be expected; it is regularly in the middle and also in the initial position, for example $\square$ $=9+8$. Solving a number sentence with the frame in the initial position can be more difficult, so this is often reserved for Challenge sections or to add an extra dimension to a set of familiar facts.

## Operations symbols ( $=/+/$ ) and language

In Number Facts 3 the equals sign is typically towards the end of the number sentence. In Number Facts 4 it is also regularly located towards the beginning of the number sentence, for example $24=4 \times$ $\square$ . This is to encourage children to become familiar with this positioning and to reinforce the idea that both sides of the equals sign should be the same (balanced).

Symbols like,,$+- x$ and $\div$ can be very abstract. This is why the initial part of every unit uses words instead of symbols, both to promote language development alongside the development of thinking strategies and to reduce the potential for misunderstanding. Furthermore, as multiplication and division are new operations in Third Class, these units are not introduced until Unit 8 in Number Facts $\mathbf{3}$ in order to allow for the children to have encountered them in their core maths programme.

|  | Operations Language |
| :--- | :--- |
| addition | add, plus, sum of |
| subtraction | subtract, minus, difference between |
| multiplication | rows of, groups of, times, double*, twos, fives, tens, etc. |
| division | share/divide into $n$ equal groups/rows <br> times smaller <br> how many groups/rows of $n$ in $\ldots ?$ <br> how many times can I take $n$ from $\ldots ?$ |

[^0]
## Flexible approach

Above all, encourage flexibility. Prompt the children to explore and discuss alternative ways to solve the questions.

## Commutative property

Children are taught to understand the commutative property of addition and multiplication (e.g. that $8+6=6+8$ and that $8 \times 6=6 \times 8$ ). In Number Facts the phrase 'turnaround facts' is often used to describe number sentences like this. However, children should also be encouraged to see that while the answers may be the same, a pictorial representation of the number sentences may be different. For example, 6 plates of 8 cookies is not the same as 8 plates of 6 cookies, but it is still 48 cookies in total. We can use the phrase same value, different appearance to explain this.

It is also important that children recognise that it can often be more efficient to think of certain facts in a certain order, e.g. $2 \times 6$ or 2 groups of $6(6+6)$ is a more efficient way to think of $6 \times 2$ or 6 groups of 2 $(2+2+2+2+2+2)$ and that facts should be manipulated in this way to their advantage where possible.

## Inverse

## Addition and subtraction

Addition and subtraction facts are taught in combined units, and it is very important that children are encouraged to appreciate how these operations are the inverse of each other.

## Multiplication and division

Multiplication and division facts are dealt with in separate units in order to break this less familiar content into more manageable weekly units. However, as with addition and subtraction, it is very important that the children develop an understanding of these operations as the inverse of each other.

- Pictorial activities, such as branching number bonds and bar models, reinforce the inverse property, as they can be recorded using multiplication and division sentences. If these are accompanied by a 'Think' box, this is usually emphasised in the text:
- The multiplication units also cover division as a missing factor, e.g. $3 \times \square=27$.

Think: 5 times what number equals 50 ?


- The branching number bonds and accompanying number sentences in the revision sections are included specifically to reinforce this concept of multiplication and division as inverse operations.


## Mental maths

Prompt the children to explore and discuss alternative ways to solve questions. For example, when asked in a Challenge section how many $€ 5$ notes equal $€ 65$, a child might suggest:

- $10 \times 5$ is 50 and then count on: $55,60,65$.
- Break 65 into the friendly numbers 50 and 15 , and work out how many fives are in each amount.

While children may suggest using a written approach, the ultimate aim is the acquisition of mental calculation skills, so encourage the children to try to use a mental method, accompanied by jottings if necessary.

## Number Talks

Number Facts is ideal for use with any class that is also participating in Number Talks. Number Talks are short, daily exercises aimed at building number sense. Children who have strong number sense can usually be flexible enough to solve calculations using a variety of thinking strategies. Number Facts greatly enhances a child's ability to verbalise and explain their strategy, even when applied to calculations outside $10 \times 10$. For more information on Number Talks please see the Weblinks document at www.edcolearning.ie. In short, the most efficient approach will always depend on the numbers involved, therefore flexibility is key.

## 3. Structure of the Books

The Number Facts series comprises weekly units that include activities for Monday to Thursday. These activities may be completed during maths class or assigned as homework activities. The sequence of the weekly units in each Number Facts book has been planned with due consideration to the following factors:

- There is a logical progression to the weekly units, so the series can be used in conjunction with any core maths programme.
- The series also serves as a companion to the Operation Maths programme, in that the units align (where suitable) with a relevant chapter or topic in Operation Maths (Long-term plans that include both Operation Maths and Number Facts are available to download from www.edcolearning.ie).
- The units are grouped together in logical blocks that align with the breaks that occur during the school year.
- For multi-class situations, the units in Number Facts $\mathbf{3}$ typically align with the identical units in Number Facts 4 in order to make it easier for the teacher to cover the same thinking strategy with both groups at the same time (see Long-term Plans, p. 13).
- The first multiplication facts introduced are facts of 2,10 and 5 , as these are the easiest facts for the children to internalise. These facts can then serve as benchmark numbers in order to aid the children's understanding of other groups (e.g. connections between 2,4 and 8 ; or 9 times as one set less than 10 times). This is the same approach taken in Operation Maths.
- Multiplication and division with 1 and/or 0 has been deliberately left until late in the sequence of units. While on the surface these may appear to be quite simple, research suggests that may children struggle with the zero and identity properties of multiplication and how they contrast with the similar properties of addition. It is, therefore, better not to separately focus on these concepts too early in the school year.


## Features

## Thinking strategies

Each unit has a specific underlying thinking strategy (see Thinking Strategies Overview, p. 12). The specific strategy is provided in the footer of the first page in each unit:

Thinking Strategy Add/subtract 1, 10, 100 or 1,000 by changing the relevant digit by 1 .
It is recommended that the thinking strategy is emphasised as often as possible during the unit. However, it is also important to encourage the children to consider or suggest if a different thinking strategy could also have been used, as this promotes flexible thinking.

## Think boxes

Think boxes are provided (usually on the second page of each unit) that serve as

Think: why are there no $\div 0$ questions? include a visual reminder (e.g. a branching number bond, bar model or 100 Square extract); a closed question (e.g. 'what is $4+9$ ? Now, what is $34+9$ ?') or an open question (e.g. 'could you use another thinking strategy for some of these?'). The open questions can be used to generate discussion and reflection among the class.

## Challenge section

Every day includes a Challenge section for early finishers. These sections typically include calculations that challenge, and therefore differentiate for, the high attainers to apply the strategies learned to bigger or more complex numbers. It is not expected that all children will be able to complete
(3) Challenge
(a) $450+\square=900$
(b) $\square+150=800$
(c) $900-350=\square$
(d) $800-250=$
the challenge sections; however, they should all be encouraged to attempt some or all of the questions. Furthermore, a quick whole-class activity on a subsequent day to share the children's approaches may generate valuable discussion and insights.

## Number limits

In the addition and subtraction units, the Challenge questions often involve regrouping and/or numbers beyond the traditional limits of $10+10$. In the multiplication and division units, the Challenge questions provide children with opportunities to apply the specific thinking strategy of the unit to quantities outside the usual range for mental recall, i.e. up to $10 \times 10$.

While these calculations may be outside of the usual range for mental recall of number facts, for the most part they are still within the limits the curriculum specifies for the class (see Maths Content Objectives, pages 14 and 15 ).
In a small number of cases, calculations have been included that go beyond these limits, e.g. in Number Facts 4, Unit 7 includes bonds of 10,000; and Unit 29 requires children to find one-seventh of an amount. These have been deliberately included, as the operations required are still within the class limits, and the purpose is to challenge the higher attainers to apply the strategies learned to more complex calculations.

## Alternative approaches

Encourage the children to look for alternative and less common ways to solve the calculations in the Challenge sections, in particular for multiplication involving two-digit numbers or more, and not to rely solely on traditional written approaches.

One strategy is to encourage the children to look out for ways to use the answers to simpler calculations to solve more complex calculations, i.e. to use groups of related facts to solve calculation where possible.

$$
\begin{aligned}
& 10 \times 9=90 \text {, so } 20 \times 9=180 \text { and } 30 \times 9=270 . \\
& 5 \times 9=45 \text {, so } 15 \times 9=135(90+45) \text {, and } 25 \times 9=225(180+45) .
\end{aligned}
$$

In particular, many of the challenge sections in Number Facts 4 have been specifically designed to encourage children to solve the questions in this way. Sets of related calculations are often referred to as number strings, and are often included as the basis for Number Talks (for more on Number Talks see p. 8).

## Communication

It is recommended that the children are allowed to present the approaches they took to solve the Challenge questions. While, as adults, we might have our own ideas of the 'right' way to solve these, the children should be encourage to explore their own methods and to explain these to the class. For example, based on the questions to the right, some alternative


Treble what number is 48 ? methods that might be suggested include:

- process of elimination: $3 \times 10=30$, which is too small; $3 \times 20=60$, which is too big; $3 \times 15=45$, which is very close, but slightly too small, so the answer is likely to be $3 \times 16$.
- compatible amounts: the children can use a branching bond to partition the number into more compatible amounts: 48 can be split into $30+18$; treble 10 is 30 and treble 6 is 18 , so the answer is $10+6=16$.
- related facts: $3 \times 10=30$, so $3 \times 15=45(30+15)$, so $3 \times 16=48(30+18)$.
- familiar facts: $48=6 \times 8$, and also $3 \times 16$ (doubling and halving).


## Family card games

A selection of card games is provided for parents and children to play at home in order to reinforce the number facts taught in each unit. The suggested card game for each unit is included in the footer of the second page in each unit, as a 'Dear Family' note:

The games are explained on page 70 of the Number Facts books. Card games specifically were included, as a pack of cards is the most likely resource that a family would have at home. However, if these games are being played at school, they could be easily adapted for use with dice, e.g. roll the dice and double the number for Doubles Snap; roll the dice and multiply by 5 for Times Snap with 5, etc.

## Revision and Assessment

## Self-assessment

The children can assess their own learning at the end of every week by ticking a box to represent their experience of that unit:


## Revision

A Revision section is included at the back of each Number Facts book. There are five Revision sections, which usually comprise ten number branching bonds. The children must complete each bond and then write four number sentences to match each bond. This reinforces the fact that four different number sentences can be written for each set of related numbers. The page footers indicate when children are ready to complete the relevant Revision section:

```
You are now ready to do Revision A (page 58)
```


## Assessment

An Assessment section is included at the back of each Number Facts book. There are seven Assessments, which comprise four sets of ten questions (number sentences). There is a note included after each Revision section that directs the children to the relevant Assessment:

```
You are now ready to do Assessment A (page 66).
```

The children can keep track of their assessment scores in their Personal Progress Chart on page 69 of each book.


Note: the Revision and Assessment activities do not have to be administered specifically at the time instructed in the footers (as outlined above); they can be administered at any stage after the relevant content has been covered. Assessments F and G are general assessments for completion at any point after the book content has been completed.

## Early finishers

If this book is intended for school rather than home use, it can be useful to have ideas ready for the early finishers:

- These children could be asked to write number sentences into their copies to match the pictorial elements included in that unit, such as branching bonds or bar models.
- Story time: pick out a number sentence on the page, e.g. $5 \times 6=\square$, and make up a story (word problem) to suit the number sentence. For example, 'There are 5 plates with 6 cookies on each. How many cookies is that altogether?'
- They could play the suggested Family Card Game in class.


## 4. Thinking Strategies Overview

A thinking strategy is a way to think about a process to arrive efficiently at an answer. In Number Facts 3 and Number Facts 4 the focus will largely be on thinking strategies for multiplication and division. It is important that children realise that there can often be many different ways to think about the same fact, so they should be encouraged both to identify alternative approaches and to choose their preferred strategy. The thinking strategies that children will learn in these books include:
(a) Multiplication as groups or rows of a number The children are enabled to recognise that ' $6 \times 2$ ' means 6 groups/rows of 2 , or $2+2+2+2+2+2$.
(c) Division as the inverse of multiplication Knowing the answer to $2 \times 6$ and $6 \times 2$ enables the children to solve $12 \div 6$ and $12 \div 2$.
(e) Multiplying and dividing with 2

- To multiply with 2 , double the number.
- To divide by 2, halve the number.


## (g) Multiplying and dividing with 4

- To multiply with 4 , double the number and then double the result, e.g. $3 \times 4 \Rightarrow 3 \times 2=6 \Rightarrow 6 \times 2=12$.
- To divide by 4 , halve the number and then halve the result, e.g.
$12 \div 4 \Rightarrow \frac{1}{2}$ of $12=6 \Rightarrow \frac{1}{2}$ of $6=3$.
(b) Turnaround facts The answer to $6 \times 2$ is the same as the answer to $2 \times 6$, and it is more efficient to think of $2 \times 6$, i.e. 2 groups of 6 or $6+6$.
(d) Multiplying and dividing with 10 and 100 The children are enabled to multiply and divide with 10 and 100 by moving the digits one/two places.
(f) Multiplying with 5 Multiply the number by 10 and then halve the result, e.g.

$$
9 \times 5 \Rightarrow 9 \times 10=90 \Rightarrow \frac{1}{2} \text { of } 90=45 .
$$

## (h) Multiplying and dividing with 8

- To multiply with 8 , double the number, double the result, and then double again, e.g.

$$
3 \times 8 \Rightarrow 3 \times 2=6 \Rightarrow 6 \times 2=12
$$

$$
\Rightarrow 12 \times 2=24
$$

- To divide by 8 , halve the number, halve the result, and then halve again, e.g.

$$
\begin{aligned}
24 \div 8 & \Rightarrow \frac{1}{2} \text { of } 24=12 \Rightarrow \frac{1}{2} \text { of } 12=6 \\
& \Rightarrow \frac{1}{2} \text { of } 6=3 .
\end{aligned}
$$

(i) Multiplying with 1 and 0 When multiplying a number by 1 , the answer is always the same as the number. When multiplying any number by zero, the answer is always zero.

## (k) Multiplying with 6

- Double 3 times the number, e.g. $6 \times 8=(3 \times 8)+(3 \times 8)=48$.
- Add one set to 5 times the number, e.g. $6 \times 8=(5 \times 8)+(1 \times 8)=48$.
(j) Multiplying with 3 The children are enabled to treble the number, or to add one set to 2 times the number, e.g.

$$
3 \times 8=(2 \times 8)+(1 \times 8)=24
$$

## (I) Multiplying with 9

- Treble 3 times the number, e.g. $9 \times 8=(3 \times 8)+(3 \times 8)+(3 \times 8)=72$.
- Subtract one set from 10 times the number, e.g. $9 \times 8 \Rightarrow(10 \times 8)=80 \Rightarrow 80-8=72$.

It is important that children realise that there can often be many different ways to think about the same fact, so they should be encouraged both to identify alternative approaches and to choose their preferred strategy.
For example, consider $5 \times 9$ :

5 times is half of 10 times: $10 \times 9=90$, so $5 \times 9=$ half of $90=45$.
9 times is one set less than 10 times: $10 \times 5=50$, so $9 \times 5=50-5=45$.
9 times is treble 3 times: $3 \times 5=15$, so $9 \times 5=$ treble $15=45$.


[^0]:    * The term 'double' is used both to mean $n+n$ and $2 \times n$.

